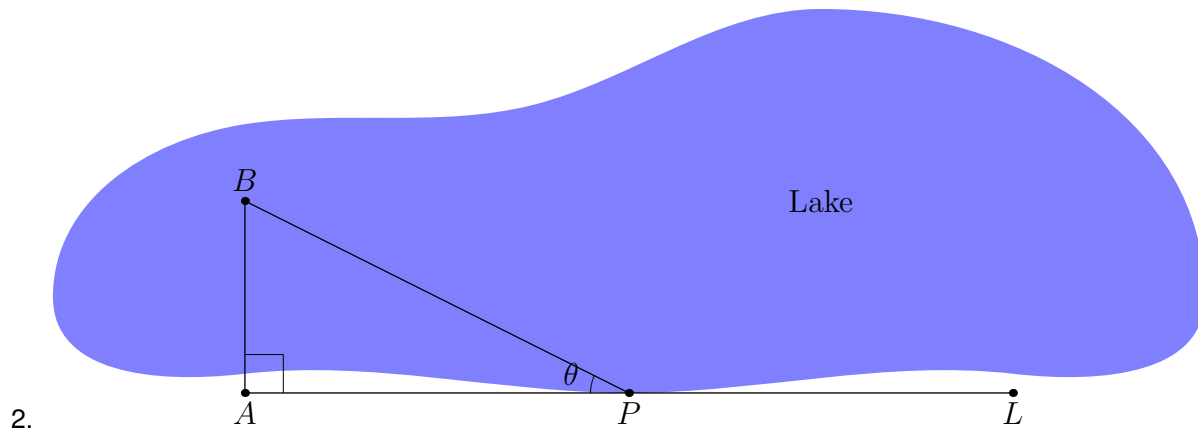


Practice Final Exam  
Calculus I

1. A hemisphere of radius 1 sits on a horizontal plane. A cylinder stands with its axis vertical, the center of its base at the center of the sphere, and its top circular rim touching the hemisphere. Find the radius and height of the cylinder of maximum volume.



A pigeon is released from a boat (point  $B$ ) floating on a lake. Because of falling air over the cool water, the energy required to fly one meter over the lake is twice the corresponding energy  $e$  required for flying over the bank ( $e = 3$  joule/meter). To minimize the energy required to fly from  $B$  to the loft,  $L$ , the pigeon heads to a point  $P$  on the bank and then flies along the bank to  $L$ . The distance  $\overline{AL}$  is 2000m and  $\overline{AB}$  is 500m.

a. Express the energy required to fly from  $B$  to  $L$  via  $P$  as a function of the angle  $\theta$ .

b. What is the optimal angle  $\theta$ ?

3. A dose,  $D$ , of a drug causes a temperature change,  $T$ , in a patient. For  $C$  a positive constant,  $T$  is given by

$$T = \left( \frac{C}{2} - \frac{D}{3} \right) D^3$$

What is the rate of change of temperature change with respect to dose?

4. Calculate  $\lim_{x \rightarrow 0} \frac{\sin x}{x^2}$ .

5. Calculate  $\lim_{x \rightarrow -\infty} (e^x)^{x^2}$ .

6. Calculate  $\lim_{x \rightarrow \frac{\pi}{2}} \tan x$ .

7. Calculate  $\lim_{x \rightarrow 0^+} \left( \ln x - \frac{1}{x} \right)$ .

8. Calculate  $\lim_{x \rightarrow 3^+} \frac{x^3 |2x - 6|}{x - 3}$

9. On what interval(s) is the following function differentiable?

$$f(x) = \begin{cases} 1 & \text{if } x \leq 1 \\ x & \text{if } 1 < x < 2 \\ -x^2 + 5x - 4 & \text{if } x \geq 2 \end{cases}$$

10. Evaluate  $\frac{d}{dx} \arccos \ln x^2$ .

11. Evaluate  $\frac{d}{dx} \frac{x^{\sin x}}{\sec x}$ .

12.  $x^2 + y^2 + y = \cos x$ . Solve for  $\frac{dy}{dx}$ .

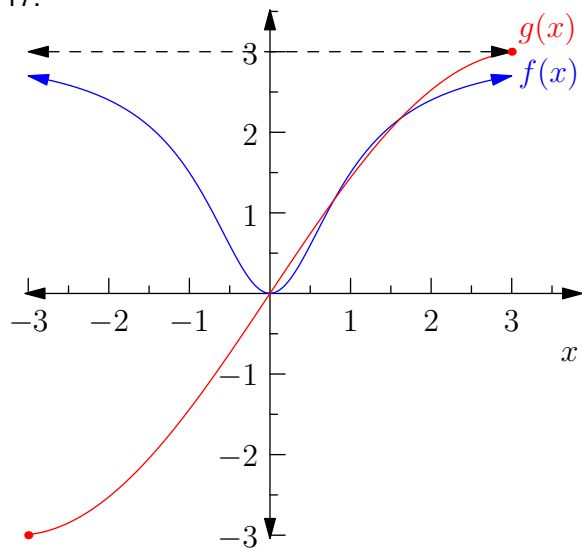
13. Calculate  $\int_2^{-1} (2x^3 + 4x^2 - 1)dx$ .

14. Calculate  $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{-1}{\sqrt{1-x^2}} dx$ .

15.  $f(x)$  is an even function.  $\int_{-2}^0 f(x)dx = 1$  and  $\int_0^1 f(x)dx = 2$ . Calculate  $\int_1^2 f(x)dx$ .

16. Write the equation of a function that has a global minimum at  $x=-1$  and an inflection point at  $x=2$ .

17.



Put the following values in order from least to greatest.

$$\int_{-1}^1 g(x)dx \quad \int_0^{\frac{1}{2}} [g(x) - f(x)]dx \quad \int_{-\frac{1}{2}}^0 f(x)dx \quad f'(-1) \quad g'(0)$$

18. A baseball thrown directly upward at 96 ft/sec has velocity  $v(t) = 96 - 32t$  ft/sec at time  $t$  seconds. Graph the velocity from  $t = 0$  to  $t = 6$ , shade an area that represents how high the baseball is at time  $t = 5$ , and estimate this height.

19. Evaluate  $\int \frac{1 - x \sin x}{x} dx$

20. Sketch an antiderivative of the function

